# Chapter 7

## Interferometry And Thin Films

## Outline

- ***Wavefront*** division interferometers
- ***Amplitude*** division interferometers
- Thin-film interference
- Thin material films; Antireflective coatings
- Thin-film filters

- Explore several important ways in which the interference process is used in today's technology.
- Young's experimental configuration represents one class of interferometer, the ***wavefront division interferometer***, in which two parts of a wavefront are caused to interfere with one another.
- In a second class, the ***amplitude-division interferometer***, the wavefront is separated into two parts by the provision of two paths for the whole wavefront.

# Wavefront-Division Interferometers

- Young's apparatus represent the typical form of a wavefront-division interferometer.
- Another important example of this kind of interferometer is the ***Rayleigh interferometer***, which is used to ***measure the index of refraction of air and other gases***.
- Fig. 7.1 shows its design.

- The source wave is divided into two parts, one passes through tube $T_1$, and the other passes through $T_2$.
- One tube is ***evacuated***, and the other is ***filled with the gas under study***.
- $\lambda_0$: wavelength in the evacuated tube; $\lambda$: wavelength in the gas-filled tube
- Recall Eq. (1-1) and using Eq. (6-7), $n = \frac{c}{v} = \frac{f\lambda_0}{f\lambda} = \frac{\lambda_0}{\lambda}, \Rightarrow \lambda = \frac{\lambda_0}{n}$

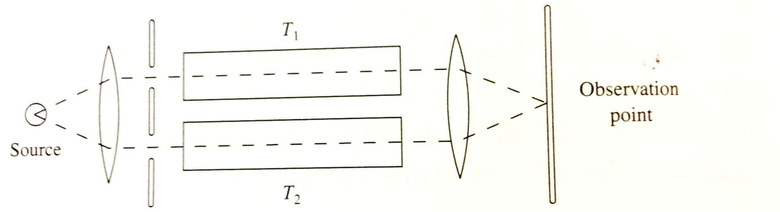



**FIGURE 7.1.** The Rayleigh interferometer. Tube $T_1$ is filled with gas, and tube $T_2$ is evacuated. The difference in path length is used to find the index of refraction of the gas in $T_1$.

- If the length of both tubes is $L$, then the number of waves in the gas-filled tube is $$N_1 = \frac{L}{\lambda} = \frac{nL}{\lambda_0}.$$
- And the number of waves in the evacuated tube is $N_2 = \frac{L}{\lambda_0}.$
- Optical path = $n$ x $L$
- Difference in optical path length: measured by the number of waves along the path,

$$N_1 - N_2 = \frac{nL}{\lambda_0} - \frac{L}{\lambda_0} = (n-1)\frac{L}{\lambda_0}.$$

- The filled tube is evacuated and the ***number of fringes passing the observation point*** are counted: $N_1$-$N_2$

# Example 7.1

- What is the difference in wavelength of 600-nm light in air as compared with the free-space (vacuum) value? $n_{air}$=1.000285 at room temperature.

# Example 7.2

- A Rayleigh interferometer is used to find the index of refraction of ether (醚). The tubes of the interferometer are evacuated, and ether gas is allowed to enter tube $T_1$. The source is a helium-neon laser operating at 623.8nm, and the tubes are each 10.15 cm long. As the ether enters the tube, the observer counts fringes and finds 243 fringes. The observer then counts the fringes passing the observation point as the tube is evacuated and again finds 243. What is the index of refraction of ether gas?

# Figure 7.2

- The use of two slits is ***not*** the most effective procedure for finding the wavelength.
- Consider the case of ***three*** slits.
- When the additional distance from slit 3 to the screen is ***one*** $\lambda$ longer than that from slit 1, the wave will ***constructively*** interfere; however, the path from slit 2 will be ***$\lambda/2$ longer*** than that from slit 1, so that paths 1 and 2 will ***destructively*** interfere and only light from slit 3 will illuminate the screen at that point.
- There will be a ***secondary maximum*** with an amplitude 1/3 that of the central maximum and an intensity of ***1/9*** that of central maximum

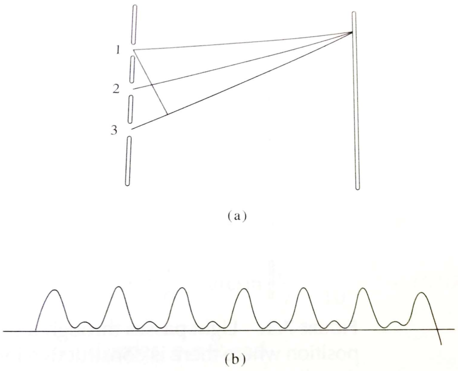



FIGURE 7.2. (a) A three-slit experiment showing the additional path lengths from slits 2 and 3 compared with that from slit 1. (b) A portion of the interference pattern that appears on the screen, showing the presence of the secondary maxima.

- When the additional path length from slit 3 is ***$2\lambda$ longer***, that from slit 2 is ***$\lambda$ longer***, and the three paths interfere ***constructively***, giving a pattern with a maximum nine times that of the secondaries, as shown in the figure.
- This concept can be extended to ***more*** slits.
- In general, the number of secondary maxima will be (**$N$-2**), and the relative intensity of the secondary maxima will be **$1/N^2$**.

# Figure 7.3

- A multislit system: light passes through each of the slits, and interfere occurs at angles where the difference in path length between the successive slits is a multiple of the wavelength.
- If $d$ is the spacing between slits, and $\lambda$ is the wavelength of source, then the angle $\theta$ at which constructive interference will occur is given by

$$\sin\theta = \frac{n\lambda}{d}$$

where $n$ is an integer 1, 2, 3, …

- The relationship is shown in Fig. 7.4.

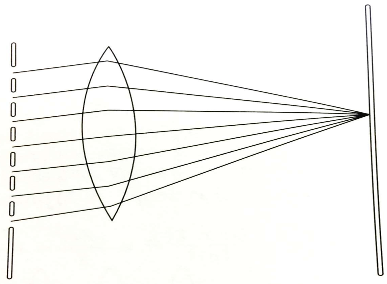


**FIGURE 7.3.** Light passes through a series of slits. The position where there is constructive interference by all the slit sources appears as a bright fringe on the screen.

# Figure 7.4

$$d\sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{d}$$

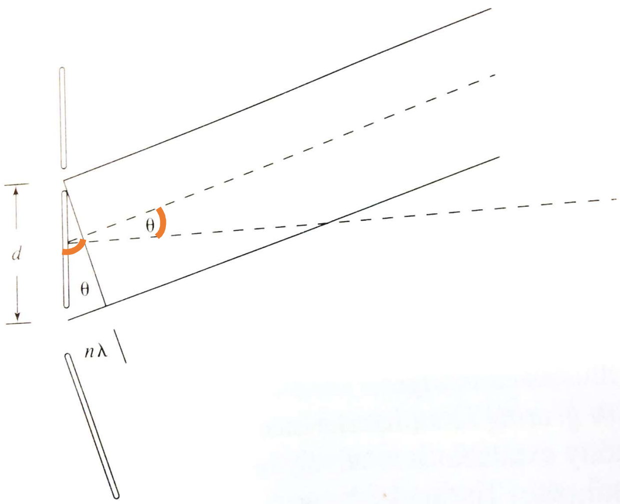



**FIGURE 7.4.** The geometry of two adjacent slits in a multiple-slit interference process.

- A ***grating*** is an interference device with ***a large number of slits***.
- The angles between the central maximum and the position of the maximum associated with a particular wavelength are easily measured and can be measured at either side of the central maximum.
- One important factor in such measurement is the ***narrow form*** of the interference line.
- With gratings the maxima obtained are very sharp as a results of the narrowness of the individual slits, so that ***very accurate measurement*** of the wavelength of a particular source can be obtained.

# Example 7.3

- White light is incident on a grating with 5000 lines/cm. At what angle are the first- and second-order red lines for a source wavelength of 650 nm found?

# Example 7.4

- In order to separate a laser beam into three beams with a 45$^o$ separation, a grating is to be blazed (閃耀). What should the slit spacing be?

# Amplitude-Division Interferometers

- In these devices the ***entire*** wavefront is used rather than ***selected portions*** of the wavefront.
- This is usually accomplished by using ***half-silvered*** mirrors or ***pellicles*** (塑膠薄膜).
- Pellicles are thin plastic sheet stretched tightly over a frame to ensure planarity and to which a ***reflective coating*** is applied. Their reflectivity can be varied roughly from 10% to 90%.

# Michelson Interferometer

- The source illuminates the lens $L_1$ to provide a collimated beam of light.
- The light falls on the half-silvered mirror, which divided the amplitude into two parts.
- One part travels up to a fixed ***full-silvered*** mirror through ***a compensation plate*** whose purpose is to make the two arms of the interferometer equal.
- Compensating plate: made of glass with the same index of refraction and thickness as the mirror but with no coating.
- In the second arm, the full-silvered mirror is mounted on a ***Vernier screw*** which has a micrometer scale that allows the mirror to be moved through an accurately measured path
- Both beams are recombined at the half-silvered mirror and are focused by lens $L_2$ on the detector.

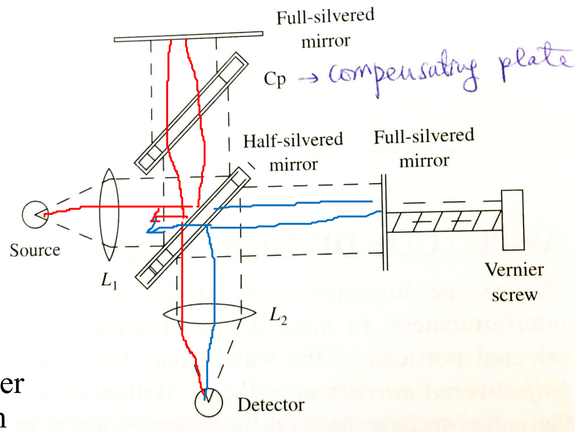



FIGURE 7.5. The Michelson interferometer design.

# Figure 7.6

- When the full-silvered mirrors are set perpendicular to the beams in their respective arms, the observer at the detector sees a ***series of circular fringes***.
- As the Vernier screw is advanced so that the relative length of two arms change, ***the central circular fringe will go from dark to bright to dark***, and so on.
- Each time the length of the variable arm is changed by λ /2, the length of the path in arm changes by λ, and the fringes go from bright to bright or dark to dark.
- By advancing the mirror in the moveable arm and measuring the change on the Vernier screw micrometer attached to the screw, one can, for example, ***find the wavelength of source***.

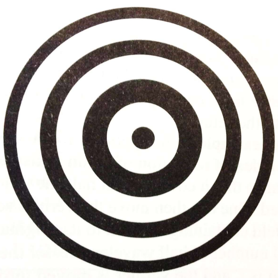


FIGURE 7.6. A series of circular fringes such as one might observe in an aligned Michelson interferometer.

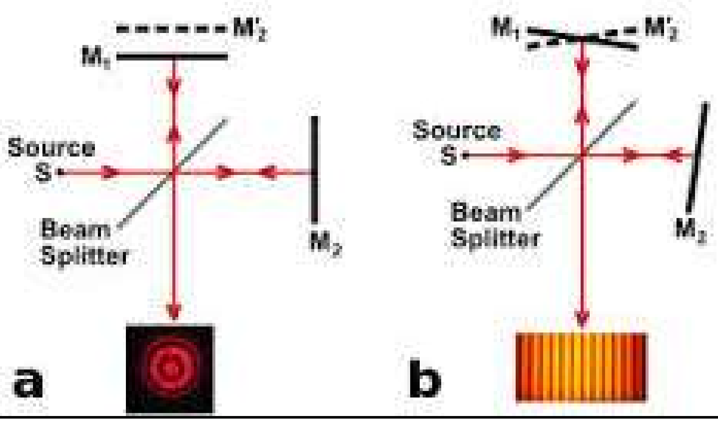
# Michelson Interferometer Demo on Yutube

- https://www.youtube.com/watch?v=j-u3IEgcTiQ

# Example 7.5

- A helium-neon laser is used as the source in the Michelson interferometer. The observer finds that moving the moveable mirror through a distance of 0.07975 mm causes 252 bright fringes to pass the observing point. What is the wavelength of the laser?

- By setting one of the mirrors at a small angle to the axis of the arm, the center of the circular interference pattern is well moved off to one side.
- Nearly straight fringes are observed → much easier to be counted!




# Figure 7.7

- Another example of the application of the Michelson interferometer is in evaluating the ***index of refraction of a thin transparent sheet***.
- If the film thickness is $t$, and the Vernier is moved through a distance $d$ to restore the original fringe position, then the index can be calculated from the equation

$$d = t(n-1)$$

where $n$ is the refractive index of the film.

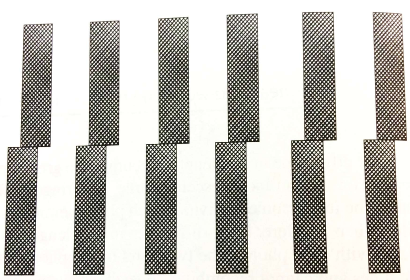
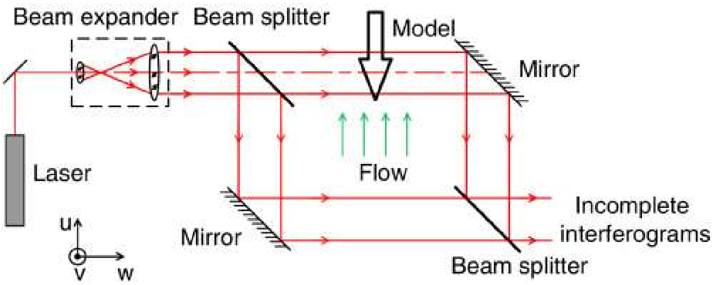


FIGURE 7.7. Fringes with a thin transparent sheet covering half the field in one arm of a Michelson interferometer.

# Mach-Zehnder Interferometer

- A second example of amplitude division interferometer
- One arm functions as a ***reference*** beam and the second as the ***experimental*** arm.



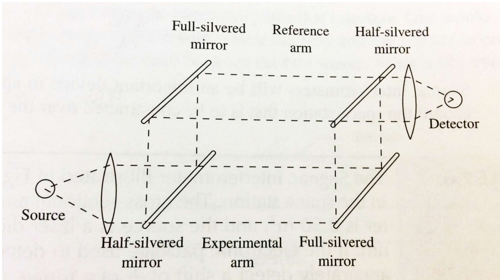




**FIGURE 7.8.** The structure of the Mach–Zender interferometer.

# Mach-Zehnder Interferometer Youtube Demo

- https://www.youtube.com/watch?v=M6y_igUpyCg&index=14&list=PL91bz_mExTfLlrG31yMEGpX7pqxj1-csw

# Figure 7.9 Sagnac Interferometer

- A third form of ***amplitude-division*** interferometer
- Used as a rotational sensor in inertial (慣性的) guidance devices in much the same way that gyroscopes (陀螺儀) are used.
- If $A$ is the cross-sectional area of the device, ω the angular velocity about the rotation center, $c$ the velocity of light, and λ the wavelength of the monochromatic source, then the shift in the number of fringes Δ observed is

$$\Delta = \frac{4A\omega}{c\lambda}$$

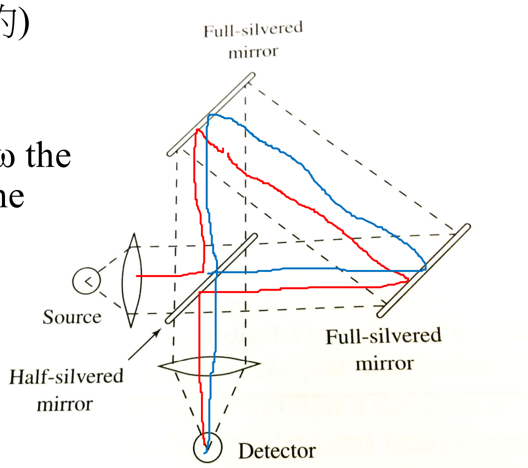



**FIGURE 7.9.** The Sagnac interferometer.

# Sagnac Interferometer Youtube Demo

- https://www.youtube.com/watch?v=SWmlimH7laY

# Example 7.6

- The Sagnac interferometer illustrated in Fig. 7.9 is to be used in the space station. The cross-sectional area of the interferometer is 0.50 $m^2$, and the source is a laser diode operating at 590 nm. If the electronic package used to detect the fringe shift can accurately detect a shift of 1/20, what is the minimum rotational velocity of the station that the interferometer can measure?

# Thin-Film Interference

- An important measurement technique
- One of the most important uses → the measurement of the ***curvature*** of a lens surface
- Fig. 7.10: a lens whose curvature is to be measured is placed on an optical flat, which is an accurately ground optical surface (~1/50 λ)
- When the lens is observed from above in monochromatic light, interference rings are seen similar to the circular fringes.
- These rings, called ***Newton's rings***, are a result of the air film between the flat and the lens.

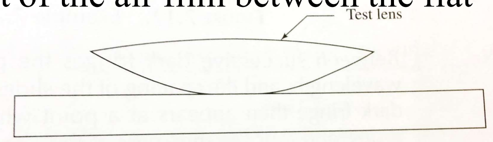



FIGURE 7.10. The lens to be tested rests on an accurately ground optical flat.

# Fig. 7.11 – Two wave paths that interfere

- https://www.youtube.com/watch?v=PU-SeNfIRcs

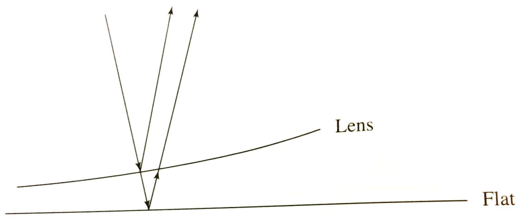



FIGURE 7.11. The two wave paths in the formation of Newton's rings.

# Example 7.7

- Two microscope slides are used to evaluate the thickness of an ultrathin wire. The experimental arrangement is as shown in Fig. 7.12, with the light source being a green laser at 550 nm. The contact between the two slides is of course a dark fringe, and at the position of the wire the eighteenth dark fringe appears. What is the thickness of the wire?

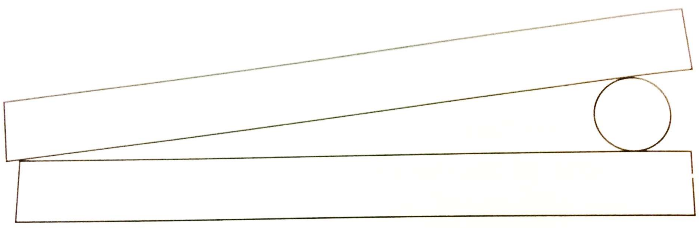


FIGURE 7.12. Example 7.7.

- Used to find radius of curvature of a lens
- The spacing between a spherical surface and a plane as shown in Fig. 7.13 is given by the sagittal theorem:

$$d = \frac{x^2}{2R}$$

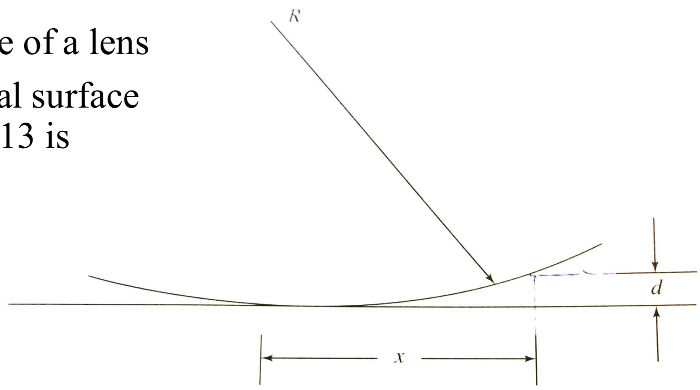



FIGURE 7.13. The sagittal depth is $d$ at a distance $x$ from the point of contact of the lens and the flat for a lens surface of radius $R$.

# Example 7.8

- A convex lens is placed on an optical flat and illuminated from above by 590 nm yellow light from a sodium source. The fifth dark fringe occurs at a distance of 16.3 mm from the center. What is the radius of curvature of the surface?

# Thin Material Films; Antireflective Coatings

- The interference exhibited by thin films can be used to eliminate or enhance reflectivity.
- In case of a lens system with a large number of surface, the loss in light energy can be substantial.
- If the reflected light was to be reduced in some fashion, then this loss could be minimized.
- Reflected light is eliminated by the use of thin-film coatings on the surface.
- The thickness of the thin film is adjusted so that there is destructive interference in the reflected light.
- Because energy must be conserved, the light that is not reflected will be transmitted.

- Vapor-phase deposition: the coating material is evaporated onto the surface of the lens in a vacuum chamber
- Two coating materials are magnesium fluoride, $MgF_2$, with an index of $n$=1.38, and cryolite, with $n$=1.36.
- For complete interference the refractive index of the coating should be the geometric mean of the index of the glass and air (1.61 for flint glass).
- $MgF_2$ and cryolite are among the best materials currently available.

- With $MgF_2$ and cryolite both the air-coating and the coating-glass reflections are rate to dense, and the thickness of the coating *t* for minimal reflection is

$$t = (2m+1)\frac{\lambda_0}{4n_c}$$

where $m$=0, 1, 2, 3, …

- The condition for minimal reflection is that odd multiples of a quarter wavelength fall within the coating.

# Example 7.9

- What is the thickness of a $MgF_2$ thin-film coating on crown glass with n=1.38 that will give minimal; reflectance for 550 nm light?

## Thin-Film Filters

- Quarter-wave stacks can be used to form the high-reflectance mirrors required for lasers, color-selective bandpass filters, and narrow-pass filters.
- These stacks are composed of alternating high- and low-index materials.
- Each layer consists of a quarter-wave thickness of each material, and as the number of layers increases, the reflectivity at that wavelength goes up.
- Sufficiently layers, often as many as 100, give a very narrow band of reflectivity with a reflectance approaching 99.5% as well as minimal absorbency.

## Homework

- Problems 2,3,5,8,11